Chapter 6: Fluid Dynamics

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CHAPTER 6

FLUID DYNAMICS

FLUID:
A substance that can flow is called fluid.
e.g.: Liquids, Gases.

FLUID STATICS:
The study of properties of fluids
at rest is known as Fluid Statics.
It is based on Newton’s first and third law.

FLUID DYNAMICS:
The study of properties of
fluids in motion is known as Fluid Dynamics.
The study of fluids in motion is
relatively complicated, but analysis can be simplified
by the use of Newton’s laws and conservation of
mass and conservation of energy.

- The law of conservation of mass gives us the
equation of continuity.
- The law of conservation of energy is the
basis of Bernoulli’s theorem.

6.1 Viscous Drag And Stoke’s Law

1. Viscosity
   (a) Definition:
   The internal frictional effect
   between different layers of a flowing fluid is
   described in terms of viscosity of the fluid.
   (b) Explanation:
   (i) Viscosity measures how much force is
   required to slide one layer of the liquid over
   another layer. Substances that do not flow easily,
such as thick tar and honey etc., have large
   coefficient of viscosity. Substances which flow
easily, like water have small coefficient of
   viscosities. Since liquids and gases have non-zero
   viscosity, a force is required if an object
   is to be moved through them.
(ii). **Example**: When we put our hand out of the window of a fast moving car, we feel that air exerts a considerable force opposite to our motion. This shows that there is a frictional force in fluids due to the viscosity of fluids.

(iii). **Symbol**: The coefficient of viscosity is denoted by greek letter ‘η’.

(iv). **Unit**: Its S.I unit is kg m\(^{-1}\) s\(^{-1}\) (i.e. \(\text{N m}^{-1}\text{m}^2\)).

(v). **Dimensions**: Its dimensions are \([M L^{-1} T^{-1}]\).

2. **Drag Force**:

An object moving through a fluid experiences a retarding force called a drag force.

- The drag force (fluid resistance) increases as the speed of the object increases.

3. **Stoke’s Law**:

The drag force \(F\) on a sphere of radius \(r\), moving slowly with speed \(v\), through a fluid of viscosity \(\eta\) is given by the Stoke’s law as

\[ F = 6\pi r \eta v \]

- At very high speeds the force is no longer simply proportional to speed.

6.2 **Terminal Velocity**

1. **Definition**: When the magnitude of the drag force becomes equal to weight of the body, the net force acting on the body is zero. Then body will fall with constant velocity called Terminal Velocity.
2. **Explanation:**

(a) Consider a water droplet such as that of fog falling vertically downward under the influence of force of gravity and drag force. The air drag on the water droplet increases with speed. The droplet accelerates rapidly under the overpowering force of gravity which pulls the droplet downward. However, the upward drag force on it increases as the speed of the droplet increases.

The net force on the droplet is:

\[ F = \text{Weight} - \text{Drag force} \]

As the speed of the droplet continues to increase, the drag force also increases and eventually approaches the weight in magnitude.

Finally, when the magnitude of the drag force becomes equal to the weight, the net force acting on the droplet is zero. Then the droplet will fall with constant velocity, called Terminal velocity \( V_t \).

To find the terminal velocity \( V_t \) in this case, we use Stokes Law for the drag. Equating it to the weight of the droplet:

\[ W = mg = \frac{4}{3} \pi r^3 \eta V_t \]

Equation (2) shows that the terminal velocity of an object depends upon mass of the object if radius remains constant, i.e., \( V_t \propto m \), i.e., the massive objects of same radius fall faster through the same fluids.

(b) **Dependence on Size**

Let \( \rho \) be the density and \( V \) be the volume of fog (water droplet), then
The mass of the droplet \( m = \rho V \)
where volume of spherical droplet \( V = \frac{4}{3} \pi r^3 \)
\( m = \frac{4}{3} \pi \rho r^3 \rho \)
Putting this value in equation (2)
\[ V_t = \frac{4 \rho \pi r^3 \rho g}{6 \pi \eta r} \]
\[ V_t = \frac{2 \rho \pi r^2}{9 \eta} \]
\( \frac{2 \rho \pi r^2}{9 \eta} = \text{Const.} \)

**Example 6.1:** A tiny water droplet of radius 0.010 cm descends through air from a high building. Calculate its terminal velocity. Given that \( \eta \) for air = 19 x 10\(^6\) kg m\(^{-1}\) s\(^{-1}\) and density of water \( \rho = 10^3 \) kg m\(^{-3}\).

**Solution:**
\( r = 0.010 \text{ cm} = 0.010 \times 10^{-2} \text{ m} = 1.0 \times 10^{-4} \text{ m} \)
\( \rho = 1000 \text{ kg m}^{-3} \)
\( \eta = 19 \times 10^6 \text{ kg m}^{-1} \text{ s}^{-1} \)

As we have
\[ V_t = \frac{2 \rho \pi r^2}{9 \eta} \]
\[ = \frac{2 \times 9.8 \times 1000 \times (1.0 \times 10^{-4})^2}{9 \times 19 \times 10^6} \text{ m}^{-1} \]
\[ V_t = 1.1 \text{ m s}^{-1} \]

Can you do that? A table tennis ball can be made suspended in the stream of air coming from the nozzles of a hair dryer. When weight of the ball is balanced by the force produced by the hair dryer i.e. \( F = W \)
6.3 Fluid Flow

Introduction: Moving fluids are of great importance. To study the behaviour of the fluid in motion, we consider their flow through the pipes. When a fluid is in motion, its flow can be either

- Streamline / Laminar
- Turbulent

1. Streamline or Laminar Flow

(a). Definition: The flow is said to be streamline or laminar, if every particle that passes a particular point, moves along exactly the same path as followed by particles which passed that point earlier.

(b). Explanation:

![Streamline Flow](image)

Fig. (a)

(i) Line of Flow:
The path followed by a particle of the fluid is called line of flow.

(ii) Stream Line:
If each particle of the fluid moves along a smooth path, then line of flow is called a streamline as shown in figure (a).

(iii). Steady Flow Condition:
If the different streamlines cannot cross each other. This condition is called Steady Flow Condition as shown in fig. (a).

- The direction of streamlines is the same as the direction of the velocity of fluid at that point.

(c). Examples:
(i). Dolphins have streamlined bodies to assist their movement in water.
(ii). Formula One racing cars have a streamlined design.
2. **TURBULENT FLOW:**

(a) **Introduction:** Above a certain velocity of the fluid flow, the motion of the fluid becomes unsteady and irregular.

(b) **Definition:** The irregular or unsteady flow of the fluid is called turbulent flow.

(c) **Explanation:** Under this condition (unsteady or irregular flow) the velocity of the fluid changes abruptly as shown in Fig (b).

In this case, the exact path of the particles of the fluid can not be predicted.

**6.4 EQUATION OF CONTINUITY**

(a) **Definition:** For a non-viscous and an incompressible fluid in steady flow, the net rate of flow of mass inward across any closed surface is equal to the net rate of flow of mass outward. This is the statement of equation of continuity.

(b) **Expression:** Consider a fluid flowing through a pipe of non-uniform size. The particles in the fluid move.
along the streamlines in a steady state flow

as shown in figure.

In the small time \( \Delta t \), the fluid at the lower end of the tube moves a distance \( \Delta x_1 \), with velocity \( V_1 \). If \( A_1 \) is the area of cross section of this end, then

\[ \text{Volume of fluid in lower shaded region} = A_1 \Delta x_1 \]

Let density of fluid be \( \rho \). Therefore the mass of the fluid contained in lower shaded region is:

\[ \Delta m_1 = \rho A_1 \Delta x_1 \quad (\text{mass} = \text{density} \cdot \text{volume}) \]

As \( s = vt \)

So \( \Delta x_1 = V_1 \Delta t \)

\[ \therefore \Delta m_1 = \rho A_1 V_1 \Delta t \]

Similarly, the fluid that moves with velocity \( V_2 \) through the upper end of the pipe (area of cross section \( A_2 \)) in the same time \( \Delta t \) has mass:

\[ \Delta m_2 = \rho A_2 V_2 \Delta t \]

If the fluid is incompressible and the flow is steady, the mass of the fluid is conserved. That is, the mass that flows into the bottom of the pipe through \( A_1 \) in a time \( \Delta t \) must be equal to the mass of the liquid that flows out through \( A_2 \) in the same time.

Therefore

\[ \Delta m_1 = \Delta m_2 \]

or

\[ \int A_1 V_1 \Delta t = \int A_2 V_2 \Delta t \]

This equation is called the equation of continuity.

Since density is constant for the steady flow of incompressible fluid, the equation of continuity becomes:
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\[ \Delta A \cdot V_1 = \Delta A_2 \cdot V_2 \]
\[ \therefore \frac{\Delta A_1}{\Delta A_2} = \frac{V_2}{V_1} \]

This equation is another form of equation of continuity. This equation shows that
1. The product of cross sectional area of the pipe and the fluid speed at any point along the pipe is a constant. This constant equals the volume flow per second of the fluid or simply 
flow rate.
2. \[ AV = \text{Constant} = \text{Volume flow rate} \]

**NOTE:**
\[ V = \text{Const.} \]
\[ A \]
\[ \rightarrow V \propto \frac{1}{A} \]

In steady incompressible flow the speed of flow varies inversely with the cross sectional area.

**Tip:**
As water falls, its speed increases due to force of gravity and so its cross sectional area decreases as mandated by the continuity equation.

**Example:** 6.2: A water hose with an internal diameter of 20 mm at the outlet discharges 30 kg of water in 60 s. Calculate the water speed at the outlet. Assume the density of water is 1000 kg m\(^{-3}\) and its flow is steady.

**Solution:**
Mass flow per second = \[ \frac{\Delta m}{\Delta t} = \frac{30}{60} = 0.5 \text{ Kg s}^{-1} \]

Cross sectional area = \( A = \pi \times 10^{-4} \text{ m}^2 \)

\[ A \]

As \( \Delta m = \Delta A \cdot V \) at
then \( \frac{\Delta m}{\Delta t} = \frac{\Delta A \cdot V}{\Delta t} \)
\[ V = \frac{\Delta m}{\Delta A \cdot \Delta t} \]

Putting values in eqn. 1
\[ V = \frac{0.5 \text{ Kg s}^{-1}}{1000 \text{ Kg m}^3 \times 3.14 \times (0.01 \text{ m})^2} \]
\[ V = 1.6 \text{ m s}^{-1} \]
6.5 *BERNOULLI’S EQUATION*

1. **Introduction:** Bernoulli’s Equation, named after a Swiss physicist Daniel Bernoulli, who discovered it, is the fundamental equation in fluid dynamics that relates pressure to fluid speed and height.

2. **Statement:** It is defined as:

   For an incompressible non-viscous fluid under going steady flow, the sum of pressure, kinetic energy per unit volume, and potential energy per unit volume remains constant at all points on a streamline.

3. **Explanation:** In deriving Bernoulli’s equation, we assume that the fluid is incompressible, non-viscous and flows in a steady state manner.

Let us consider the flow of the fluid through the pipe in time $t$, as shown in figure.

As $P - P_0$, the force on the upper end of the fluid is $\Delta F$, where $P$ is the pressure and $P_0$ is the area of cross section at the upper end. The work done on the fluid, by the fluid behind it in moving
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It through a distance \( \Delta x_1 \), will be

\[
W_1 = \frac{F_1 \cdot \overrightarrow{d}}{V} = F_1 \cdot \Delta x_1 \cdot \cos \theta = P_1 A_1 \Delta x_1
\]

Similarly, the work done on the fluid at the lower end is

\[
W_2 = - F_2 \Delta x_2 = - P_2 A_2 \Delta x_2
\]

where \( P_1 \) is the pressure, \( A_1 \) is the area of cross-section of the lower end, and \( \Delta x_2 \) is the distance moved by the fluid in the same time interval \( t \). The work \( W_2 \) is taken to be negative as this work is done against the fluid force exerted by the liquid which is right of (ahead of) the lower shaded water as shown in the figure.

The net work done

\[
W = W_1 + W_2
\]

\[
W = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2
\]

\[ v_1 \text{ and } v_2 \text{ are the velocities at the upper and lower ends respectively, then} \]

\[ \Delta x_1 = v_1 t \text{ and } \Delta x_2 = v_2 t \]

\[
W = P_1 A_1 v_1 t - P_2 A_2 v_2 t
\]

From equation of continuity

\[ A_1 v_1 = A_2 v_2 = \text{Volume flow per second} = \frac{V}{t} \]

Hence

\[ A_1 v_1 t = A_2 v_2 t = V \text{ (volume)} \]

So we have

\[
W = P_1 V - P_2 V
\]

\[
W = (P_1 - P_2) V
\]

If \( m \) is the mass and \( \rho \) is the density of water, then

\[ V = \frac{m}{\rho} \]

So equation \( 3 \) becomes

\[
W = (P_1 - P_2) \frac{m}{\rho}
\]
Part of this work is utilized by the fluid in changing its K.E. and a part is used in changing its gravitational P.E.

\[
\text{Change in K.E.} = \Delta (K.E.) = \frac{1}{2} m V_1^2 - \frac{1}{2} m V_2^2
\]

\[
\text{Change in P.E.} = \Delta (P.E.) = mg h_2 - mg h_1
\]

where \(h_1\) and \(h_2\) are the heights of the upper and lower ends respectively.

Applying law of conservation of energy to this volume of the fluid, we get

\[
\text{Work} = \Delta K.E. + \Delta P.E.
\]

\[
(P_1 - P_2) \frac{m}{p} = \frac{1}{2} m V_1^2 - \frac{1}{2} m V_2^2 + mg h_2 - mg h_1
\]

\[
(P_1 - P_2) \frac{m}{p} = \frac{1}{2} V_1^2 - \frac{1}{2} V_2^2 + gh_2 - gh_1
\]

\[
P_1 - P_2 = \frac{1}{2} \rho V_1^2 - \frac{1}{2} \rho V_2^2 + \rho g h_2 - \rho g h_1
\]

Re-arranging the equation

\[
P_1 + \frac{1}{2} \rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g h_2
\]

This is Bernoulli's equation and can be expressed as

\[
P + \frac{1}{2} \rho V^2 + \rho g h = \text{Const.}
\]

i.e.

\[
P + \frac{1}{2} \rho \frac{V^2}{V} + \rho g \frac{h}{V} = \text{Const.}
\]
APPLICATIONS OF BERNOULLI’S EQUATION

1. TORRICELLI’S THEOREM:

A simple application of Bernoulli’s equation is shown in figure.

Suppose a large tank of fluid has two small orifices A and B, unit as shown in figure.

Let us find the speed with which the water flows from the orifice A.

Since the orifices are so small, the efflux speeds $v_2$ and $v_3$ will be much larger than the speed $v_1$ of the top surface of water, i.e., $v_2$ and $v_3 > v_1$.

We can therefore take $v_1$ as approximately zero.

As Bernoulli’s equation

$$ P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 $$

As $v_1 = 0 \implies P_1 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$

But $P_1 = P_2 =$ atmospheric pressure

So $R + \rho g h_1 = R + \frac{1}{2} \rho v_2^2 + \rho g h_2$

$$ \rho g h_1 = \frac{1}{2} \rho v_2^2 + \rho g h_2 $$

$$ v_2 = \sqrt{2g(h_1 - h_2)} $$

This is Torricelli’s theorem which states $\sqrt{2as} = v_2$.

The speed of efflux is equal to the velocity gained by the fluid in falling through the distance $(h_1 - h_2)$ under the action of gravity.
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2. Relation Between Speed and Pressure of the Fluid:

A result of the Bernoulli’s equation is that the pressure will be low where the speed of the fluid is high.

Suppose that water flows through a pipe system as shown in the figure. The water will flow faster at B than at A or C. Assuming the flow speed at A to be 0.20 m/s and at B to be 2.0 m/s, we compare the pressure at B with that at A.

Applying Bernoulli’s equation

\[ p_a + \frac{1}{2} \rho V_a^2 + \frac{1}{2} \rho g h_a = p_b + \frac{1}{2} \rho V_b^2 + \frac{1}{2} \rho g h_b \]

But the average P.E. is the same at both places, i.e.

\[ \frac{1}{2} \rho g h_a = \frac{1}{2} \rho g h_b \]

\[ \therefore p_a + \frac{1}{2} \rho V_a^2 = p_b + \frac{1}{2} \rho V_b^2 \]

Putting \( V_a = 0.20 \text{ m/s} \), \( V_b = 2.0 \text{ m/s} \) and \( \rho = 1000 \text{ kg/m}^3 \).
we get \[ P_a - P_b = 1980 \text{ N m}^2 \]

This shows that the pressure in the narrow pipe where streamlines are closer together is much smaller than in wider pipe.

**Conclusion:**

Where the speed is high, the pressure will be low.

**Examples:**

(a) The lift on an aeroplane is due to this effect. The flow of air around an aeroplane wing is illustrated in figure. The wing is designed to deflect the air so that streamlines are closer together above the wing than below it. It can be seen in figure that where the streamlines are forced closer together, the speed is faster. Thus, air is travelling faster on the upper side of the wing than on the lower. The pressure will be lower at the top of the wing, and the wing will be forced upward.

(b) When a tennis ball is hit by a racket in such a way that it spins as well as moves forward, the velocity of the air on one side of the ball increases due to higher pressure and air speed in the same direction increases so pressure decreases. This gives an extra curvature to ball known as swing.
3. VENTURI RELATION:

If one of the pipes has a much smaller diameter than the other as shown in figure.

Applying Bernoulli’s equation.

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \]

As \( \rho g h_1 = \rho g h_2 \) \((\because h_1 = h_2)\)

\[ P_1 - P_2 = \frac{1}{2} \rho v_1^2 - \frac{1}{2} \rho v_2^2 = \frac{1}{2} \rho (v_1^2 - v_2^2) \]

As the cross-sectional area \( A_2 \) is small as compared to the area \( A_1 \), then from equation of continuity

\[ v_1 = \left( \frac{A_2}{A_1} \right) v_2 \]

\[ v_1 < v_2 \], thus for flow from a large pipe to a small pipe we can neglect \( v_1 \) on the right hand of the equation.

\[ \therefore v_1 < v_2 \] \( \Rightarrow \) \( v_1 \to 0 \)

\[ P_1 - P_2 = \frac{1}{2} \rho v_2^2 \]

This relation is known as Venturi relation which is used in Venturi meter, a device used to measure speed of liquid flow.
(4) Blood Flow

Blood is an incompressible fluid having a density nearly equal to that of water. A high concentration (~50%) of red blood cells increases its viscosity from three to five times that of water. Blood vessels are not rigid. They stretch like a rubber hose. Under normal circumstances, the volume of the blood is sufficient to keep the vessels inflated at all times, even in the relaxed states between heart beats. This means there is tension in the walls of the blood vessels and consequently the pressure of blood inside is greater than the external atmospheric pressure.

Figure (b) shows the variation in blood pressure as the heart beats.

For Healthy Person: The pressure varies from a high (systolic pressure) of 120 torr to a low (diastolic pressure) of about 75-80 torr between beats in normal healthy person. The numbers tend
to increase with age, corresponding to the decrease in the flexibility of the vessel walls.

Torricelli’s Law: The unit torr or mm of Hg is opted instead of SI unit of pressure because of its extensive use in medical equipments.

1 torr = 133.3 N/m²

Sphygmomanometer: It is an instrument which measures blood pressure dynamically.

Measurement of Blood Pressure:

An inflatable bag is wound around the arm of a patient and external pressure on the arm is increased by inflating the bag. The effect is to squeeze the arm and compress the blood vessels inside. When the external pressure applied becomes larger than the systolic pressure, the vessels collapse, cutting off the flow of the blood. Opening the release valve on the bag gradually decreases the external pressure.

A stethoscope detects the instant at which the external pressure becomes equal to the systolic pressure. At this point, the first surge of blood flows through the narrow structure produces a high flow speed. As a result, the flow is initially turbulent.

As the pressure drops, the external pressure eventually equals the diastolic pressure. From this point, the vessel no longer collapse during any portion of the flow cycle. The flow switches from turbulent to laminar, and the gurgle in the stethoscope disappears. This is the signal to record diastolic pressure.
EXAMPLE 6.3: Water flows down through a closed vertical funnel. The flow speed at the top is 12 cm/s. The flow speed at the bottom is twice the speed at the top. If the funnel is 40 cm long and the pressure at the top is 1.013 x 10^5 N m^-2, what is pressure at the bottom?

**Solution:**

Speed of water at top: \( v_1 = 12 \text{ cm/s} = 0.12 \text{ m/s} \)

Speed of water at bottom: \( v_2 = 2v_1 = 0.24 \text{ m/s} \)

Density of water: \( \rho = 1000 \text{ Kg/m}^3 \)

Length of funnel: \( h = h_1 - h_2 = 40 \text{ cm} = 0.4 \text{ m} \)

Using Bernoulli's Equation:

\[
P_1 + \rho g h_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2} \rho v_2^2
\]

\[
P_2 = P_1 + \rho g h - \rho g h_2 + \frac{1}{2} \rho (v_1^2 - v_2^2)
\]

\[
= P_1 + \rho g h + \frac{1}{2} \rho (v_1^2 - v_2^2)
\]

\[
= (1.013 \times 10^5 \text{ N/m}^2) + (1000 \text{ Kg/m}^3 \times 9.8 \text{ m/s}^2 \times 0.4 \text{ m})
\]

\[
+ \frac{1}{2} (1000 \text{ Kg/m}^3) \left( (0.12 \text{ m/s})^2 - (0.24 \text{ m/s})^2 \right)
\]

\[
P_2 = 1.05 \times 10^5 \text{ N/m}^2
\]

Answer.
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EXERCISE

SHORT QUESTIONS

Q. 6.1: Explain what do you understand by the term viscosity?

Ans.: See article 6.1 in theory.

Q. 6.2: What is meant by drag force? What are the factors upon which drag force acting upon a small sphere of radius \( r \) moving down through a fluid depend?

Ans.: Drag Force: An object moving through a fluid experience a retarding force (which acts opposite to its motion) is called Drag Force.

By stoke's law its relation is given as:

\[
F_d = 6\pi r \eta v
\]

Factors:

(i) Velocity: The drag force is directly proportional to the speed of the body in the fluid.

(ii) Radius: The drag force also depends upon radius of the object (sphere).

(iii) Nature of fluid: It also depends upon coefficient of viscosity of the fluid, i.e. nature of the fluid.

Note: In the given case, radius of small sphere is given i.e. \( r \) which means that radius remains constant. Then drag force will depend upon velocity of the object and nature of the fluid only.

Also \( V_t = \frac{2gr^2}{\eta} \). So \( V_t \propto g \).

Q. 6.3: Why fog droplets appear to be suspended in air?

Ans.: As we know that

\[
V_t = \frac{mg}{6\pi r^3 \eta}
\]

as mass of fog droplet is very small, so its terminal velocity will also be very small.
For this reason, fog droplets appear to be suspended in air as its speed is very low.

6.6.4. Explain the difference between laminar flow and turbulent flows.

Ans. See Theory.

6.6.5. State Bernoulli’s relation for a liquid in motion and describe some of its applications.

Ans. See Theory.

6.6.6. A person is standing near a fast moving train. Is there any danger that he will fall towards it?

Ans. As we know from applications of Bernoulli’s theorem that, where the speed is faster, there the streamlines are forced closer together and pressure will be low there.

So between train and person streamlines are closer due to high speed, there will be low pressure exerted by fluid on person. But on the other side speed of fluid is low and streamlines are spaced together so from that side pressure will be greater. Therefore, there will be a danger for person that he will fall towards the fast moving train.

6.6.7. Identify the correct answer. What do you infer from Bernoulli’s theorem?

(i) Where the speed of the fluid is high, the pressure will be low?
(ii) Where the speed of the fluid is high, the pressure is also high.

(iii) This theorem is valid only for turbulent flow of the liquid.

Ans. The correct answer is (i).
Where the speed of the fluid is high, the pressure will be low.

Q. 6.8: Two row boats moving parallel in the same direction are pulled towards each other. Explain.

Ans. According to Torricelli's theorem and relation b/w velocity and pressure, the stream lines are forced closer with each other if speed is faster. And where speed is faster, the pressure will be lower.
Between row boats, the velocity of fluid is high but pressure is low and outside the boats velocity is low but pressure is large, so these pressures will force the boats towards each other as shown in figure.

Q. 6.9: Explain how the swing is produced in a fast moving cricket ball.

Ans. When a bowler bowled a cricket ball, then it spins as well as moves forward. The velocity of air on one side of the ball increases on one of its surface (i.e. Shinnedone) due to which pressure decreases.
whereas on the other surface (i.e. rough one), velocity of air decreases, so pressure increases. This gives an extra curvature to ball known as ‘wobble’ which may deceive an opponent batsman.

Q. 6.10:- Explain the working of a carburetor of a motorcar using Bernoulli’s principle.

Ans: The carburetor of a car engine uses a Venturi duct to feed the correct mix of air and petrol to the cylinders. Air is drawn through duct and along a pipe to the cylinders. A tiny inlet at the side of duct is fed with petrol.

The air through the duct moves very fast, creating low pressure in the duct, which draws petrol vapour into the air stream.

Q. 6.11:- For which position will the maximum blood pressure in the body have the smallest value?

(a) Standing up right  (b) Sitting  (c) Lying horizontally  (d) Standing on one’s head.

Ans: (c) The maximum blood pressure in the body have the smallest value when body is lying horizontally.
Q. 6.12. In an orbiting space station, would the blood pressure in major arteries in the leg ever be greater than the blood pressure in major arteries in the neck?
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PROBLEMS

P. 6.1: A certain globular protein particle has a density of 1246 kg m\(^{-3}\). It falls through pure water (\( \eta = 8.0 \times 10^{-4} \text{ Nm} \text{s}^{-2} \)) with a terminal speed of 3.0 cm h\(^{-1}\). Find the radius of particle.

Solution:

Data:

- Density of protein particle: \( \rho = 1246 \text{ kg m}^{-3} \)
- Coefficient of viscosity of water: \( \eta = 8.0 \times 10^{-4} \text{ Nm} \text{s}^{-2} \)
- Terminal velocity \( V_t = 3 \text{ cm h}^{-1} = \frac{3 \times 10^{-2}}{3600} \text{ m s}^{-1} \)
- Terminal velocity \( V_t = 8.33 \times 10^{-6} \text{ m s}^{-1} \)

Calculations:

As we have

\[
V_t = \frac{2 \eta r^2 g}{9 \eta}
\]
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\[ r^2 = \frac{Q}{2\eta} \]

\[ r^2 = \frac{9 \times 8 \times 10^{-4} \text{ Nm}^2 \times 8.33 \times 10^6 \text{ m}^3}{2 \times 9.8 \text{ m}^3 \times 1246 \text{ Kg m}^{-3}} \]

\[ r = \sqrt{25 \times 10^{-10}} \text{ m} \]

Answer: \[5 \times 10^{-5} \text{ m} \]

P. 6.2: Water flows through a hose, whose internal diameter is 1 cm, at a speed of 1 m/s. What should be the diameter of the nozzle if the water is to emerge at 2 m/s?

Solution

Data: \[ d_1 = 1 \text{ cm} = 10^{-2} \text{ m} \]

Speed of water flow \( v_1 = 1 \text{ m/s} \)

Speed of water emergence \( v_2 = 2 \text{ m/s} \)

Diameter of the nozzle \( d_2 = ? \)

Calculation:

According to equation of continuity

\[ A_1 v_1 = A_2 v_2 \]  \( \text{(1)} \)

where \[ A = \pi r^2 \]

So

\[ A_1 = \pi r_1^2 = \pi \left( \frac{d_1}{2} \right)^2 \]  \( (\because d = 2r) \)

\[ A_1 = \pi d_1^2 \frac{d_1}{4} \]

Similarly

\[ A_2 = \pi d_2^2 \frac{d_2}{4} \]

Putting in eq. \( \text{(1)} \)

\[ \pi \frac{d_1^2}{4} v_1 = \pi \frac{d_2^2}{4} v_2 \]
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\[ d_1^2 v_1 = d_2^2 v_2 \]
\[ d_2^2 = \frac{d_1^2 v_1}{v_2} \]
\[ d_2 = \sqrt{\frac{v_1}{v_2}} d_1 \]
\[ = \sqrt{\frac{1}{21} \times (0.1)^2} = \sqrt{0.05} \times (0.1) \]
\[ = 0.002 \text{ meters} \]
\[ d_2 = 0.2 \text{ cm} \quad \text{Answer} \]

P. 6.3 : The pipe near the lower end of a large water storage tank develops a small leak and a stream of water shoots from it. The top of water in the tank is 15 m above the point of leak.

(a). With what speed does the water rush from the hole?

(b). If the hole has an area of 0.060 cm², how much water flows out in one second?

**Solution:**

**Data:**
- Height of water = \( \Delta h = 15 \text{ m} \)
- Area of hole = \( A = 0.06 \text{ cm}^2 \)
- Speed of water emergence = \( V = ? \)
- Rate of water emergence = \( R = ? \)

**Calculations:**

(a). According to Torricelli’s theorem

\[ V = \sqrt{2g \Delta h} \]
\[ \quad = \sqrt{2 \times 9.8 \times 15} \text{ m/s} \]
\[ V = 17.14 \text{ m/s} \quad \text{Answer} \]

(b). From equation of continuity

Volume flow rate = \( A \cdot v \)
$v = 17.14 \text{ m/s} = 1714 \text{ cm/s}$

Volume flow rate = \(1714 \text{ cm}^3/\text{s} \times 0.06 \text{ cm}^2\)

\[= 102 \text{ cm}^3/\text{s}\]

So volume of water flows out in one second

\[V = 102 \text{ cm}^3\] Answer

Problem 6.4: Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3 m/s, while at another point, 3 m higher, the speed is 4.0 m/s. If the pressure is 80 kPa at the lower point, what is the pressure at the upper point?

Solution:

Data:

- Speed of water on lower end = \(V_1 = 3 \text{ m/s}\)
- Speed of water on upper end = \(V_2 = 4 \text{ m/s}\)
- Pressure at lower end = \(P_1 = 80 \text{ kPa} = 80 \times 10^3 \text{ Pa} = 80 \times 10^3 \text{ N/m}^2\)
- Height of water = \(\Delta h = h_2 - h_1 = 3 \text{ m}\)

Calculations:

According to Bernoulli's equation,

\[P_1 + \frac{1}{2} \rho V_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho V_2^2 + \rho g h_2\]

or

\[P_2 = P_1 + \frac{1}{2} \rho (V_1^2 - V_2^2) + \rho g (h_1 - h_2)\]

\[= 80 \times 10^3 \text{ Pa} + \frac{1}{2} \times 1000(3^2 - 4^2) + (1000 \times 9.8 \times 3)\]

\[= 47100 \text{ Pa}\]

\[= 47.10 \times 10^3 \text{ Pa}\]

\[P_2 = 47 \text{ kPa}\] Answer
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P. 6.5: An aeroplane wing is designed so that when the speed of the air across the top of the wing is 450 m/s, the speed of air below the wing is 410 m/s. What is the pressure difference between the top and bottom of the wings?

**SOLUTION:**

**DATA:**
- Speed of air on top \( v_1 = 450 \text{ m/s} \)
- Speed of air below \( v_2 = 410 \text{ m/s} \)
- Density of air \( \rho = 1.29 \text{ kg/m}^3 \)
- Difference in pressure \( \Delta P = ? \)

**CALCULATION:**
According to Bernoulli’s relation,
\[
\frac{P_2}{\rho} - \frac{P_1}{\rho} = \frac{1}{2} \rho (v_1^2 - v_2^2)
\]
\[
\Delta P = \frac{1}{2} \rho (v_1^2 - v_2^2)
\]
\[
= \frac{1}{2} \times 1.29 \times (450)^2 - (410)^2 \text{ Pa}
\]
\[
= \frac{1}{2} \times 1.29 \times 34400 \text{ Pa}
\]
\[
= 22.188 \times 10^3 \text{ Pa}
\]

\[\Delta P = 22 \times 10^3 \text{ Pa} \]

Answer: \( \Delta P = 22 \times 10^3 \text{ Pa} \)

P. 6.6: The radius of the aorta is about 1.0 cm and the blood flowing through it has a speed of about 30 cm/s. Calculate the average speed of the blood in the capillaries using the fact that although each capillary has a diameter of about 8x10^-4 cm, there are literally millions of them so that their total cross section is about 2000 cm^2.
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Data:
- Radius of aorta: \( r_1 = 1 \text{ cm} \)
- Speed of blood: \( V_1 = 30 \text{ cm}^2\text{s}^{-1} \)
- Diameter of capillary: \( d = 8 \times 10^{-4} \text{ cm} \)
- Radius of capillary: \( r = 4 \times 10^{-4} \text{ cm} \)
- Total area of cross section of capillaries: \( A_2 = 2000 \text{ cm}^2 \)
- Average speed of blood: \( V_2 = ? \)

Calculations:
- Area of aorta: \( A_1 = \pi r_1^2 = 3.14 \times (1 \text{ cm})^2 = 3 \text{ cm}^2 \)
- From equation of continuity
  \[ A_1 \cdot V_1 = A_2 \cdot V_2 \]
  \[ V_2 = \frac{A_1}{A_2} \cdot V_1 = \frac{3 \text{ cm}^2 \times 30 \text{ cm}^2\text{s}^{-1}}{2000 \text{ cm}^2} = 4.7 \times 10^{-2} \text{ cm}^2\text{s}^{-1} \]
  \[ 4.7 \times 10^{-4} \text{ m}^2\text{s}^{-1} \]
  \[ V_2 = 5 \times 10^{-4} \text{ m}^2\text{s}^{-1} \]
  Answer

P 6.7: How large must a heating duct be if air moving 3.0 m\(^3\text{s}^{-1}\) along it can replenish the air in a room of 300 m\(^3\) volume every 15 min? Assume the air's density remains constant.

Solution:
Data:
- Speed of air: \( V = 3 \text{ m}\text{s}^{-1} \)
- Volume of air: \( V = 300 \text{ m}^3 \)
- Time: \( t = 15 \text{ min} = 15 \times 60 = 900 \text{ sec} \)
- Size of duct: \( r = ? \)
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CALCULATIONS:

According to equation of continuity

\[ A \nu = \text{Volume flow rate} \]
\[ A \nu = \frac{V}{t} \]
\[ \pi r^2 \nu = \frac{V}{t} \quad (\because A = \pi r^2) \]
\[ r^2 = \frac{V}{\pi \nu t} \]
\[ r = \sqrt{\frac{V}{\pi \nu t}} = \sqrt{\frac{300 \text{ m}^3}{3.14 \times 3 \text{ m}^2 \times 900 \text{ s}}} \]

\[ r = 0.19 \text{ m} = 19 \text{ cm} \]

P. 6.8: An airplane design calls for a "lift" due to the net force of the moving air on the wing, of about 1000 Nm² of wing area.

Assume that air flows past the wing of an aircraft with streamline flow. The speed of flow past the lower wing surface is 160 m/s. What is the required speed over the upper surface to give a lift of 1000 Nm²? The density of air is 1.29 kg/m³ and assume max. thickness of wing be one meter

SOLUTION:

Data:
- Pressure on wing, \( \Delta P = 1000 \text{ N/m}² \)
- Speed of air, \( v_i = 160 \text{ m/s} \)
- Density of air, \( \rho = 1.29 \text{ kg/m}³ \)
- Thickness of wing, \( t = 1 \text{ m} \)
- Speed of uplift, \( v_u = ? \)

Calculation:

According to "Venturi" relation

\[ \Delta P = \frac{1}{2} \rho (v_u^2 - v_i^2) \]

\[ \Delta P = \frac{1}{2} \rho (v_u^2 - v_i^2) \]
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\[ v_2 = \frac{2 \Delta P}{p} + v_1^2 \]

\[ v_2^2 = \frac{2 \times 1000 \text{ Nm}^{-2}}{1.29 \text{ kg m}^{-3}} + (60 \text{ m s}^{-1})^2 \]

\[ v_2 = \sqrt{\frac{2 \times 1000 \text{ Nm}^{-2}}{1.29 \text{ kg m}^{-3}} + (60 \text{ m s}^{-1})^2} \]

\[ v_2 = 164.77 \text{ m s}^{-1} \]

\[ v_2 = 165 \text{ m s}^{-1} \] **Answer**

**P. 6.9:** What gauge pressure is required in the city mains for a stream from a fire hose connected to the mains to reach a vertical height of 15 m? 

**Solution:**

**Data:**

\( \Delta h = 15 \text{ m} \)

\( g = 9.8 \text{ m s}^{-2} \)

Density of water \( \rho = 1000 \text{ kg m}^{-3} \)

\( \Delta P = ? \)

**Calculations:**

According to Bernoulli’s equation

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2 \]

or

\[ P_1 - P_2 = \rho g (h_2 - h_1) + \frac{1}{2} \rho (v_2^2 - v_1^2) \]

As speed of water throughout the flow does not change so \( v_1 = v_2 \) \( \Rightarrow v_2^2 - v_1^2 = 0 \)

So

\[ \Delta P = \rho g (h_2 - h_1) = \rho g \Delta h \]

\[ \Delta P = 1000 \times 9.8 \times 15 \text{ N m}^2 \]

\[ \Delta P = 1.47 \times 10^5 \text{ Pa} \] **Answer**